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Analytical Evaluation of Damping in Composite and Sandwich Structures

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Two methodologies are outlined for the analytical evaluation of the loss factor in composite laminates and in sandwich structures. One of these methods is based on the analysis of free vibrations, whereas the second approach utilizes mechanics of materials. Both methods yield similar results. The loss factor is predicted both for specially orthotropic as well as for generally orthotropic laminas, subjected to axial stresses and/or transverse shear. The results for the loss factor of the laminas are in good agreement with available experimental data. As follows from the numerical analysis, the loss factor of laminas of the facings is significantly affected by the angle of lamination. The loss factor of a sandwich beam is relatively insensitive to the frequency of vibrations, although the effect of this frequency on the loss factors of constituent materials may alter this conclusion.

Nomenclature

a	=	length of the beam
E_f	=	modulus of elasticity of the fibers
E_m	=	modulus of elasticity of the matrix
E_1	=	longitudinal modulus of the lamina
E_2	=	transverse modulus of the lamina
G_f	=	shear modulus of the fibers
G_m	=	shear modulus of the matrix
G_{12}	=	in-plane shear modulus of the lamina
G_{13}, G_{23}	=	transverse shear moduli of the lamina
g	=	loss factor
h	=	depth of the beam
h_c	=	depth of the core of sandwich beam
$\bar{Q}_{11}$	=	transformed reduced stiffness of the corresponding lamina
S_{ij}	=	elements of the compliance matrix
t	=	time
U	=	maximum strain energy density during the cycle
U_d	=	density of energy dissipated per cycle of motion
u_f	=	strain energy density in the fibers
u_m	=	strain energy density in the matrix
V_f	=	fiber volume fraction
V_m	=	matrix volume fraction
w	=	transverse displacement of the beam
x	=	axial coordinate
z	=	transverse (thickness) coordinate
γ	=	shear strain
ε	=	axial strain
θ	=	lamination angle
ρ	=	mass of the beam per unit length
σ	=	axial stress
τ	=	shear stress
ψ_x	=	rotation of a normal to the middle axis of the beam in the xz plane
ω	=	natural frequency

Introduction

DAMPING in composites represents an important phenomenon that has been studied by a number of investigators.[1–3] Several mechanisms contributing to damping include viscoelasticity of the constituent materials (fibers, matrix, and interphase), thermoelastic coupling, energy dissipation associated with the changes in the material stiffness as a result of internal damage, and frictional energy dissipation along damaged fiber-matrix interfaces. The relative contribution of these factors varies, dependent on a particular composite material. For example, recent research of Birman and Byrd has shown that the contribution to the loss factor of ceramic matrix composites with bridging cracks attributed to interfacial friction can exceed the counterpart related to both viscoelastic effects and a reduction in stiffness by two orders of magnitude.[4]

Although several studies on damping effects in sandwich structures have been published,[5,6] a methodology of the analytical prediction of damping in such structures has not been considered in detail. Vinson stated in his monograph[7] that sandwich structures have potential for a significant damping exceeding that in both metallic and monocoque composite structures. In the present paper the analysis is concerned with the development of a methodology for the prediction of damping in composite and sandwich structures. The solution developed in the paper refers to undamaged structures with polymeric-matrix facings and isotropic foam core. Two methods discussed in the paper are based on the free vibration analysis and on the mechanics of materials approach. For simplicity, the solution is shown for the case of a composite or sandwich beam, but the extension of the present investigation to panels and shells is straightforward.

It is useful to recall here relationships between basic characteristics of damping employed in the analysis of relatively weakly damped materials. A comprehensive discussion of the phenomenon is outside the scope of this paper; mentioned here is a useful book[8] that outlines related fundamental concepts.

The specific damping capacity of the material ψ is related to such damping characteristics as the logarithmic decrement δ, the damping ratio ζ, and the loss factor g by the following equations[1]:

$$\psi = 2\delta = 4\pi\zeta = 2\pi g \tag{1}$$

These damping characteristics are defined as functions of relative energy dissipation by

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$$g = U_d/2\pi U \tag{2}$$

The loss factor for a system of linear viscoelastic elements suggested by Ungar and Kerwin[9] is used in this paper to evaluate damping in a sandwich structure:

$$g = \sum_i g_i U_i \Big/ \sum_i U_i \tag{3}$$

where i is a number of the element, g_i is the loss factor for the ith element, and U_i is the maximum strain energy of this element during the vibration cycle.

Damping in a Generally Orthotropic Lamina Subject to Axial Stresses

Consider first a generally orthotropic lamina oriented at an angle θ relative to the longitudinal x axis and subjected to axial stresses σ_x (Fig. 1). Axial stresses can be caused by bending or axial loading. The loss factor in the case where the lamina is subject to transverse shear is considered in the next section. The principal material directions (1 and 2) are also shown in Fig. 1. The applied stress results in the stresses in the material coordinate system calculated by the transformation equation:

$$\sigma_1 = c^2\sigma_x, \qquad \sigma_2 = s^2\sigma_x, \qquad \tau_{12} = -sc\sigma_x \tag{4}$$

where $c = \cos\theta$ and $s = \sin\theta$.

The components of the strain tensor in the principal material directions are immediately available:

$$\varepsilon_1 = \left(S_{11}c^2 + S_{12}s^2\right)\sigma_x, \qquad \varepsilon_2 = \left(S_{12}c^2 + S_{22}s^2\right)\sigma_x$$

$$\gamma_{12} = -scS_{66}\sigma_x \tag{5}$$

where the elements of the compliance matrix are given in terms of the material engineering constants by $S_{11} = E_1^{-1}$, $S_{12} = -\nu_{12}E_2^{-1}$, $S_{22} = E_2^{-1}$, $S_{66} = G_{12}^{-1}$.

Now the components of the strain energy in the fibers and matrix associated with the strains in the principal material directions can be found in terms of the strains ε_1, ε_2, and γ_{12}. In particular, the fiber and matrix strain energy densities caused by the strain ε_1 are obtained keeping in mind that the strains in the 1-direction in the fibers and in the matrix are equal:

$$u'_f = \tfrac{1}{2}V_f\sigma_{1f}\varepsilon_1 = \tfrac{1}{2}V_f E_f \varepsilon_1^2$$

$$u'_m = \tfrac{1}{2}V_m\sigma_{1m}\varepsilon_1 = \tfrac{1}{2}V_m E_m \varepsilon_1^2 \tag{6}$$

The strain energy densities of the fibers and matrix in Eqs. (6) and in the following discussion are multiplied by the volume fraction of the corresponding phase. Therefore, these energy densities refer to the density of a phase within the unit volume of the composite material.

The fiber and matrix strain energy densities caused by the strain ε_2 are

$$u''_f = \tfrac{1}{2}V_f\sigma_{2f}\varepsilon_{2f} = \tfrac{1}{2}V_f E_{f2} \varepsilon_{2f}^2$$

$$u''_m = \tfrac{1}{2}V_m\sigma_{2m}\varepsilon_{2m} = \tfrac{1}{2}V_m E_m \varepsilon_{2m}^2 \tag{7}$$

where E_{f2} is a transverse modulus of the fiber material that is assumed equal to the modulus in the 1-direction [that is, E_f (isotropic fibers)].

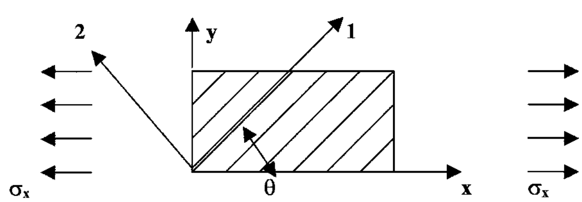

Fig. 1 Generally orthotropic lamina subject to axial stresses, θ = lamination angle, 1-2 = material coordinate system.

According to the energy approach employed by Gibson,[10] transverse strains in the fibers and in the matrix can be found as

$$\varepsilon_{2f} = a_2\varepsilon_2, \qquad \varepsilon_{2m} = b_2\varepsilon_2 \tag{8}$$

The values of constants a_2 and b_2 can be found from[10]

$$a_2V_f + b_2V_m = 1, \qquad E_2 = a_2^2E_fV_f + b_2^2E_mV_m \tag{9}$$

The fiber and matrix strain energy densities caused by the strain γ_{12} are

$$u'''_f = \tfrac{1}{2}V_f\tau_{12f}\gamma_{12f} = \tfrac{1}{2}V_f G_f \gamma_{12f}^2$$

$$u'''_m = \tfrac{1}{2}V_m\tau_{12m}\gamma_{12m} = \tfrac{1}{2}V_m G_m \gamma_{12m}^2 \tag{10}$$

The shear strains in the fibers and matrix are found similarly to the method just applied to the transverse strains:

$$\gamma_{12f} = c_2\gamma_{12}, \qquad \gamma_{12m} = d_2\gamma_{12} \tag{11}$$

where constants c_2 and d_2 are obtained from the system of equations:

$$c_2V_f + d_2V_m = 1, \qquad G_{12} = c_2^2G_fV_f + d_2^2G_mV_m \tag{12}$$

The total fiber and matrix strain energy densities are calculated as

$$u_f = (V_f/2)\left[E_f\left(\varepsilon_1^2 + a_2^2\varepsilon_2^2\right) + G_fc_2^2\gamma_{12}^2\right]$$

$$u_m = (V_m/2)\left[E_m\left(\varepsilon_1^2 + b_2^2\varepsilon_2^2\right) + G_md_2^2\gamma_{12}^2\right] \tag{13}$$

Given experimentally determined loss factors of the fiber and matrix materials, (that is, g_f and g_m) and using the total composite strain energy density equal to the sum of the contributions of the fibers and matrix, we obtain the loss factor of the lamina according to Eq. (3) as

$$g_\theta = (g_fu_f + g_mu_m)/(u_f + u_m) \tag{14}$$

Substituting Eqs. (5) into (14) and simplifying, one obtains

$$g_\theta = \left\{g_fV_f\left[E_f\left(k_1 + k_2a_2^2\right) + G_fk_3\right] + g_mV_m\left[E_m\left(k_1 + k_2b_2^2\right) + G_mk_4\right]\right\} \Big/ \left\{V_f\left[E_f\left(k_1 + k_2a_2^2\right) + G_fk_3\right] + V_m\left[E_m(k_1 + k_2b_2^2) + G_mk_4\right]\right\} \tag{15}$$

where

$$k_1 = \left(S_{11}c^2 + S_{12}s^2\right)^2, \qquad k_2 = \left(S_{12}c^2 + S_{22}s^2\right)^2$$

$$k_3 = c_2^2(scS_{66})^2, \qquad k_4 = d_2^2(scS_{66})^2 \tag{16}$$

The cases of specially orthotropic laminas corresponding to the lamination angles equal to 0 or 90 deg (that is, g_0 and g_{90}) can be obtained from the preceding result:

$$g_0 = \frac{(g_fV_fE_f/E_m + g_mV_m)}{(V_fE_f/E_m + V_m)}$$

$$g_{90} = \frac{\left(a_2^2g_fV_fE_f\big/E_m + b_2^2g_mV_m\right)}{\left(a_2^2V_fE_f\big/E_m + b_2^2V_m\right)} \tag{17}$$

The result for 0-deg layers coincides with the formula derived by Chang and Bert.[11]

Damping in a Generally Orthotropic Lamina Subject to Transverse Shear Stress (τ_{xz})

The stress has to be transformed to the principal material axes. The transformation equations of the three-dimensional theory of elasticity (for example, see p. 2 in Ref. 12) yield

$$\tau_{13} = c\tau_{xz}, \qquad \tau_{23} = -s\tau_{xz} \tag{18}$$

The strain energy densities in the isotropic fibers and matrix are

$$u_f^{ts} = (V_fG_f/2)\left(\gamma_{13f}^2 + \gamma_{23f}^2\right)$$

$$u_m^{ts} = (V_mG_m/2)\left(\gamma_{13m}^2 + \gamma_{23m}^2\right) \tag{19}$$

The transverse shear strains are obtained in the form similar to Eqs. (11):

$$\gamma_{13f} = m_2\gamma_{13}, \qquad \gamma_{13m} = n_2\gamma_{13}$$
$$\gamma_{23f} = p_2\gamma_{23}, \qquad \gamma_{23m} = q_2\gamma_{23} \tag{20}$$

where the average strains are

$$\gamma_{13} = c\tau_{xz}/G_{13}, \qquad \gamma_{23} = -s\tau_{xz}/G_{23} \tag{21}$$

and the coefficients are obtained from the systems of equations

$$m_2V_f + n_2V_m = 1, \qquad m_2^2G_fV_f + n_2^2G_mV_m = G_{13}(=G_{\mathrm{LT}}) \tag{22}$$

$$p_2V_f + q_2V_m = 1, \qquad p_2^2G_fV_f + q_2^2G_mV_m = G_{23}(=G_{\mathrm{TT}}) \tag{23}$$

Using Eqs. (19–21) and Eq. (14), one obtains the following expression for the loss factor in the case of transverse shear (no in-plane loading):

$$g_\theta^{ts} = \frac{(g_fV_fG_fk_5 + g_mV_mG_mk_6)}{(V_fG_fk_5 + V_mG_mk_6)} \tag{24}$$

where

$$k_5 = m_2^2c^2S_{55}^2 + p_2^2s^2S_{44}^2, \qquad k_6 = n_2^2c^2S_{55}^2 + q_2^2s^2S_{44}^2 \tag{25}$$

In Eqs. (25) the shear compliances are $S_{55} = G_{13}^{-1}$ and $S_{44} = G_{23}^{-1}$.

In particular cases of specially orthotropic laminas, Eq. (24) is simplified as follows:

$$g_0^{ts} = \frac{\left(g_fV_fG_fm_2^2 + g_mV_mG_mn_2^2\right)}{\left(V_fG_fm_2^2 + V_mG_mn_2^2\right)}$$

$$g_{90}^{ts} = \frac{\left(g_fV_fG_fp_2^2 + g_mV_mG_mq_2^2\right)}{\left(V_fG_fp_2^2 + V_mG_mq_2^2\right)} \tag{26}$$

It is possible to combine the preceding results for the loss factor of a generally orthotropic lamina subject to axial stresses and the loss factor in the case of transverse shear. This would require us to establish a relationship between the magnitude of the axial stress and the corresponding transverse shear stress. Consider, for example, the case where this ratio can be assumed constant during the cycle of motion (that is, $R = \tau_{xz}/\sigma_x$). Therefore, Eqs. (21) become

$$\gamma_{13} = cR\sigma_x/G_{13}, \qquad \gamma_{23} = -sR\sigma_x/G_{23} \tag{27}$$

The transverse shear strain energy density in the fibers and matrix can be determined in the form

$$u_f^{\sigma\tau} = \left(R^2V_fG_f/2\right)\left[(m_2c/G_{13})^2 + (p_2s/G_{23})^2\right]\sigma_x^2$$
$$u_m^{\sigma\tau} = \left(R^2V_mG_m/2\right)\left[(n_2c/G_{13})^2 + (q_2s/G_{23})^2\right]\sigma_x^2 \tag{28}$$

Combining this result with Eqs. (13 and 14) yields the loss factor for a lamina subject to a combined action of in-plane axial stresses and transverse shear, provided the ratio between these stresses remains constant during the cycle of motion:

$$g_{\sigma\tau} = F_1/F_2 \tag{29}$$

where

$$F_1 = g_fV_f\Big(E_f\left(k_1 + k_2a_2^2\right) + G_f\left\{k_3 + R^2\left[(m_2c/G_{13})^2 + (p_2s/G_{23})^2\right]\right\}\Big) + g_mV_m\Big(E_m\left(k_1 + k_2b_2^2\right) + G_m\left\{k_4 + R^2\left[(n_2c/G_{13})^2 + (q_2s/G_{23})^2\right]\right\}\Big)$$

$$F_2 = V_f\Big(E_f\left(k_1 + k_2a_2^2\right) + G_f\left\{k_3 + R^2\left[(m_2c/G_{13})^2 + (p_2s/G_{23})^2\right]\right\}\Big) + V_m\Big(E_m\left(k_1 + k_2b_2^2\right) + G_m\left\{k_4 + R^2\left[(n_2c/G_{13})^2 + (q_2s/G_{23})^2\right]\right\}\Big) \tag{30}$$

If the assumption that the stress ratio R remains constant during the cycle of motion is invalid, it is necessary to consider the problem of vibrations of a shear deformable facing. The analysis represents a particular case of the solution for a shear-deformable sandwich beam presented in the next section. Accordingly, the problem of determining the loss factor of a shear-deformable lamina is revisited in the following section.

Loss Factor of Vibrating Sandwich Beam: Approach Based on Free-Vibration Analysis

Consider now a simply supported unit-width sandwich beam experiencing small-amplitude free vibrations. The facings are assumed symmetric about the axis of the beam. The analysis is conducted using the following assumptions:

1) Normal mode approach is applicable in the present analysis.

2) The core works in transverse shear only, whereas the facings experience both axial strains as well as transverse shear strains.

The first assumption is customarily used in case of a relatively light damping in structures where coupling between normal modes as a result of damping can be neglected.[8] The utilization of this assumption becomes mandatory if the magnitude of damping is not known in advance. Then this assumption can serve as the first iteration (that is, damping determined utilizing the assumption can be used to recalculate the motion of the structure, which is in turn used to update the loss factor, etc).

The strain energy of the beam, measured in the units of force because it refers to the energy per unit width, is given by

$$U = \int_0^a\int_{h_c/2}^{h/2} Q_{11}\varepsilon_x^2\,\mathrm{d}z\,\mathrm{d}x + \left(\frac{A_{55}}{2}\right)\int_0^a \gamma_{xz}^2\,\mathrm{d}x \tag{31}$$

where the factor $\frac{1}{2}$ is excluded from the first term in the right side to account for the presence of two facings. The stiffness in the second term is

$$A_{55} = k\int_{-h/2}^{h/2} G_{xz}\,\mathrm{d}z \tag{32}$$

In Eq. (32) k is the shear correction factor that can be taken equal to unity,[13] and G_{xz} is the transverse shear modulus. It is convenient to subdivide the energy given by Eq. (31) into the contributions of the laminas of the facings and that of the core. The energy of the core is

$$U_c = \left(\frac{h_cG_c}{2}\right)\int_0^a \gamma_{xz}^2\,\mathrm{d}x \tag{33}$$

The energy of the ith lamina is given by

$$U_i = \left(\frac{1}{2}\right)\int_0^a\int_{z_{i-1}}^{z_i} Q_{11}^i\varepsilon_x^2\,\mathrm{d}z\,\mathrm{d}x + \left[\frac{(z_i - z_{i-1})G_{xz}^i}{2}\right]\int_0^a \gamma_{xz}^2\,\mathrm{d}x \tag{34}$$

where the superscript identifies the properties of the ith lamina and z_i and z_{i-1} are the coordinates of the lamina interfaces with adjacent laminas or with the core.

Following the first-order shear deformation theory adopted in this paper, the strains in Eqs. (33 and 34) are

$$\varepsilon_x = z\psi_{x,x}, \qquad \gamma_{xz} = w_{,x} + \psi_x \tag{35}$$

The beam being symmetric about the middle axis, this axis does not experience stretching as long as the amplitudes of motion remain small (linear vibrations).

The solution for the nth mode of free vibrations of a symmetric simply supported sandwich beam with specially orthotropic facings obtained neglecting inertia of the axial vibrations is

$$w_n = B_n\exp(-j\omega_nt)\sin(n\pi x/a)$$
$$\psi_{xn} = A_n\exp(-j\omega_nt)\cos(n\pi x/a) \tag{36}$$

where $j = (-1)^{1/2}$. Even in the case where symmetric facings include multiple generally orthotropic layers, the present solution is usually accurate because the stiffnesses D_{i6} ($i = 1, 2$) are relatively small. The ratio between the amplitudes in Eqs. (36) is

$$b_n = B_n/A_n = (n\pi/a)A_{55}\Big/\left[(n\pi/a)^2A_{55} - \rho\omega_n^2\right] \tag{37}$$

The substitution of Eqs. (36) and (37) into Eqs. (35) and the subsequent substitution of Eqs. (35) into Eqs. (2), (3), (33), and (34) yield the expression for the loss factor corresponding to the nth normal mode:

$$g_n = G_n/H_n \tag{38}$$

where

$$G_n = \left(\frac{n\pi}{a}\right)^2 \sum_i g_i D_{11}^i + \left(\sum_i g_i' h_i G_i + g_c h_c G_c\right) \times \left[\left(\frac{n\pi}{a}\right)^2 b_n^2 + 2\left(\frac{n\pi}{a}\right) b_n + 1\right]$$

$$H_n = \left(\frac{n\pi}{a}\right)^2 D_{11} + A_{55}\left[\left(\frac{n\pi}{a}\right)^2 b_n^2 + 2\left(\frac{n\pi}{a}\right) b_n + 1\right]$$

$$D_{11} = \int_{h_c/2}^{h/2} Q_{11} z^2\, dz, \qquad D_{11}^i = \int_{z_{i-1}}^{z_i} Q_{11}^i z^2\, dz$$

$$h_i = z_i - z_{i-1} \tag{39}$$

In Eqs. (39), g_i and g_i' are the loss factors of the ith lamina subjected to axial loading and transverse shear, respectively, and g_c is the loss factor of the core. Note that the loss factor of a specially orthotropic shear-deformable lamina subject to a combined action of axial loading and transverse shear can be obtained from Eqs. (38) and (39) as

$$g = \frac{(n\pi/a)^2 g_i D_{11}^i + g_i' h_i G_i\left[(n\pi/a)^2 b_n^2 + 2(n\pi/a) b_n + 1\right]}{(n\pi/a)^2 D_{11} + h_i G_i\left[(n\pi/a)^2 b_n^2 + 2(n\pi/a) b_n + 1\right]} \tag{40}$$

where the ratio b_n is calculated for the lamina material and geometry.

The loss factor given by Eq. (38) is affected by the mode and frequency. If the sandwich beam is slender (that is, if it can be analyzed by the theory of thin beams), this factor is independent of the mode and frequency, and Eq. (38) is reduced to

$$g = \left(\sum_i g_i D_{11}^i\right) \Big/ D_{11} \tag{41}$$

In the case of a symmetric slender sandwich beam where the contribution of the core can be disregarded and the facings are composed of identical "blocks" of laminas, such as for example [0 deg /+45 deg/−45 deg /90 deg]$_n$, the loss factor can be determined from a simplified version of Eq. (41). This simplified version is shown here for the case where the Poisson effect is neglected:

$$g = g_{eq} E_{eq}/E_{ave} \tag{42}$$

where

$$g_{eq} E_{eq} = \sum_i \frac{g_i E_i}{p}, \qquad E_{ave} = \sum_i \frac{E_i}{p} \tag{43}$$

and where the summation is carried out over the laminas that compose the representative block, p is a number of laminas in the block, whereas g_i and E_i are the loss factor and the modulus of the ith lamina in the beam axial direction, respectively.

Note that the loss factor of a sandwich beam depends on the boundary conditions. If these conditions differ from simple support, Eqs. (36) and (37) have to be replaced with appropriate counterparts, affecting the value of the loss factor given by Eq. (38). In addition, the loss factor is affected by the frequency of vibrations, both explicitly via the ratio b_n and implicitly because the loss factors of constituent fiber, matrix, and foam materials vary with frequency.

Loss Factor of Vibrating Sandwich Beam: Approach Based on Mechanics of Materials

The second approach considered in this paper employs simple mechanics of materials considerations. Consider a symmetrically laminated unit-width sandwich beam subject to a combination of the bending moment M and transverse shear stress resultant Q. The equilibrium equations are

$$M = D_{11}\Psi_{x,x}, \qquad Q = A_{55}\gamma_{xz} \tag{44}$$

The strain energy of the facings assumed to be in the state of plane stress and the strain energy associated with transverse shear in the facings and in the core are

$$U_f = \int_0^a \int_{h_c/2}^{h/2} Q_{11} z^2 \left(\frac{M}{D_{11}}\right)^2 dz\, dx$$

$$U_S = \left(\int_0^a \int_{-h_c/2}^{h/2} Q_{55}\gamma_{xz}^2\, dz\, dx\right)\Big/ 2 = \left(\int_0^a Q^2\, dx\right)\Big/ (2A_{55}) \tag{45}$$

where $A_{55} = \sum_i G_{xz}^i h_i + G_c h_c$.

The loss factor can be evaluated as

$$g = G/H \tag{46}$$

where

$$G = \frac{\sum g_i D_{11}^i}{D_{11}^2} \int_0^a M^2\, dx + \left[\sum \frac{g_i' Q_{55}^i h_i + g_c G_c h_c}{A_{55}^2}\right] \int_0^a Q^2\, dx$$

$$H = \frac{1}{D_{11}} \int_0^a M^2\, dx + \frac{1}{A_{55}} \int_0^a Q^2\, dx \tag{47}$$

For example, if the beam is subject to a uniform pressure of intensity q, the integrals in Eqs. (47) can easily be evaluated. In particular, the integral of the square of bending moments over the span is equal to $q^2a^5/120$, whereas the integral of the square of shear forces is $q^2a^3/12$. If we consider the case where the facings are formed from repeated identical blocks of laminas, i.e., it is possible to use an average-through-thickness loss factor and modulus of the facing, the expression for the loss factor of the sandwich beam is simplified:

$$g = \frac{\left(g_f a^2/10D_{11}\right) + \left(2g_f' G_f h_f + g_c G_c h_c\right)/A_{55}^2}{\left(a^2/10D_{11}\right) + 1/A_{55}} \tag{48}$$

In Eq. (48), g_f and g_f' are the average-through-thickness loss factors of the facing subject to axial and transverse shear stresses, respectively, and G_f is the average facing transverse shear modulus.

The loss factor given by Eq. (46) or (48) is explicitly independent of the frequency of motion because it was derived based on static mechanics of materials. However, even in this case, the loss factors of the constituent materials of the structure are affected by frequency. The solution may be expanded to analyze a distribution of moments and transverse shear forces corresponding to static loading causing deformations identical to the investigated mode of motion. In this case, the effect of the mode of motion (and explicit effect of the frequency) can be accounted for. However, such complication is unnecessary because the solution can be more accurately obtained using the vibration method shown in the preceding section. Nevertheless, the approach shown in this section can be applied to quickly estimate the loss factor of the beam. It is also noted that the loss factor obtained in this section is affected by the boundary conditions that define the distribution of moments and transverse shear forces along the beam span.

Numerical Examples

The effect of the lamination angle on the loss factor of a generally orthotropic lamina was investigated for a typical glass-epoxy material considered by Gibson and Plunkett[14] (Scotchply 1002 matrix epoxy and E-glass fibers). The properties of E-glass fibers and epoxy matrix are shown in Table 1. The material analyzed in Ref. 14 had a fairly high void volume fraction equal to 2%. Accordingly, the presence of voids is accounted for in the subsequent calculations. In addition, as was found from the comparison between numerical

Table 1 Properties of E-glass fibers and epoxy matrix

Property	E, GPa	G, GPa	Poisson ratio, ν	Volume fraction[a]
E-glass fibers	72.4	30.3	0.20	0.45
Epoxy matrix	3.79	1.38	0.36	0.53

[a]Void volume fraction is 2%.

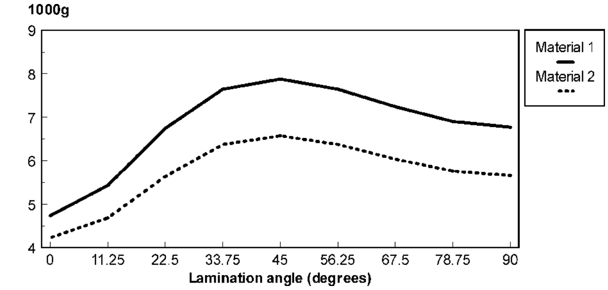

Fig. 2 Loss factor of a generally orthotropic E-glass/epoxy lamina as a function of the lamination angle. Material 1: $V_f = 0.45$, $V_m = 0.53$ (volume fraction of voids is 2%). Material 2: $V_f = 0.60$, $V_m = 0.40$.

and experimental results, the Poisson effect could be disregarded, although we do not employ this simplification in the following examples that refer to relatively wide beams.

Experimentally found values of static longitudinal and transverse moduli of the material were $E_1 = 34.8$ GPa and $E_2 = 10.5$ GPa, respectively. The latter value practically coincides with the result available using the simplified micromechanical theory of Chamis[15]:

$$E_2 = E_m \Big/ \left[1 - V_f^{\frac{1}{2}}(1 - E_m/E_f)\right] \tag{49}$$

The same theory was employed to calculate the shear modulus G_{12} that was not measured in Ref. 14. Substituting G_f and G_m instead of E_f and E_m in Eq. (49) yields $G_{12} = 3.84$ GPa.

The loss factor of the matrix was found experimentally.[14] Although this factor varied, dependent on the frequency, the low-frequency value $g_m = 0.01319$ (frequencies less than 100 Hz) was used in the analysis. Low-frequency values were also employed for other experimentally found material properties in the following examples. The loss factor of E-glass fibers was assumed negligible in Ref. 14, based on the claim of the manufacturer, although the results of the experiments caused the authors to indicate that the influence of this factor is probably noticeable. In the present paper the loss factor of E-glass fibers was estimated based on the comparison of the experimental value of the loss factor for a unidirectional composite material with the lamination angle equal to zero with the result obtained from the first Eq. (17) that yielded $g_f = 0.00346$. The loss factor obtained here for E-glass fibers is within the range of values for glass fibers reported by Lee.[16]

The values of the fiber and matrix loss factors were employed to calculate the loss factor for a 90-deg lamina that was found equal to 0.0075. This value coincides with the experimental result reported in Ref. 14.

The effect of the lamination angle on the loss factor of axially loaded laminas is shown in Fig. 2 for the material described and for the second material composed of different volume fractions of the same fibers and matrix. As follows from this figure, the loss factor of a generally orthotropic lamina can reach the maximum value at the lamination angle different from 90 deg. Note that, if the beam is narrow, i.e., the Poisson effect can be neglected, the loss factor consistently increases with the lamination angle, reaching its maximum value at the angle equal to 90 deg. (This case is not discussed here.) The loss factor of laminas subjected to transverse shear loading is less affected by the lamination angle than the factor associated with in-plane loading. This is because a difference between the shear moduli in the planes parallel to the fibers (G_{12} or G_{13}) and the modulus in the plane perpendicular to the fibers (G_{23}) is relatively small. Micromechanical equations for G_{23} obtained according to Chamis[15] indicate that this modulus remains equal to G_{13} if the materials of both fibers and matrix are isotropic. Although other theories can yield a slightly different value for G_{23} and for the loss factor, this difference remains small. In the present paper the loss factor corresponding to the case where a lamina is subject to transverse shear stresses only was $g_\theta^{ts} = 0.00804$.

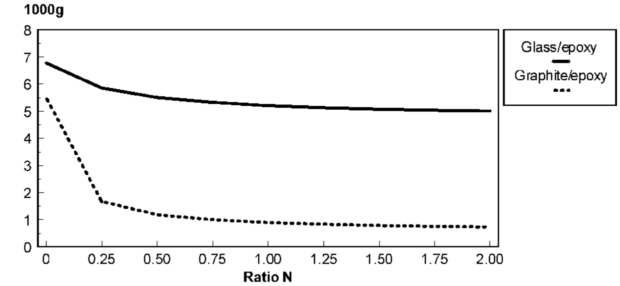

Fig. 3 Loss factor of E-glass/epoxy and graphite/epoxy cross-ply laminates as a function of the ratio of the total thickness of longitudinal layers to the total thickness of transverse layers N.

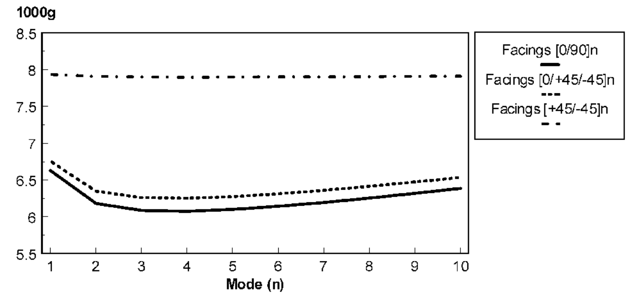

Fig. 4 Effect of the mode of vibration (vibration frequency) on the loss factor of sandwich beams with E-glass/epoxy facings of various layups.

The loss factor of a cross-ply uniaxially loaded composite material is shown in Fig. 3 as a function of the ratio of the total thickness of longitudinal layers to the total thickness of transverse layers N. Two materials compared in this figure are E-glass/epoxy just considered ($V_f = 0.45$, $V_m = 0.53$) and graphite-epoxy AS4/3501-6. The room-temperature low-frequency loss factors of the latter material are $g_0 = 0.00056$ and $g_{90} = 0.0055$ (Ref. 17). The mechanical properties of this material necessary for computations presented in Fig. 3 are $E_1 = 142.0$ GPa and $E_2 = 10.3$ GPa (Ref. 18). The loss factor calculated neglecting the Poisson effect was

$$g = (g_0 E_1 N + g_{90} E_2)/(E_1 N + E_2) \tag{50}$$

As follows from Fig. 3, the loss factor of a cross-ply material decreases as a result of adding even a few longitudinal layers to transverse laminas. As the total thickness of longitudinal layers increases, the loss factor of the material asymptotically approaches the value of this factor for the longitudinal lamina.

The effect of the mode shape of motion on the loss factor of a sandwich beam evaluated by the analysis of free vibrations is illustrated in Fig. 4. The facings of the beam are constructed of multiple identical blocks of E-glass/epoxy laminas ($V_f = 0.45$, $V_m = 0.53$), and the core properties are $G_c = 40$ MPa (as for Divinylcell foam H100) and $g_c = 0.008$. The beams considered in Fig. 4 were 0.25 m long, and the thickness of the facings and core was equal to 0.005 and 0.02 m, respectively. Although the loss factors of three different beams considered in Fig. 4 varied, the effect of the mode on these factors was small for all considered designs. This implies that variations in the loss factor with the frequency of motion are mostly due to the effect of this frequency on the loss factors of the constituent materials, particularly the matrix of the facings and the foam of the core. (This effect is not reflected in Fig. 4.)

Finally, the effect of the thickness of the facings on the loss factor of the beams considered in Fig. 4 is shown in Fig. 5 for the fundamental mode of motion (one half-wave of deformation along the beam axis). The results in Fig. 5 reflect a decrease in the contribution of the loss factor of the facings to the overall loss factor of the structure as the relative facing thickness declines. A decrease in the loss factor of the beams with thinner facings is explained by the

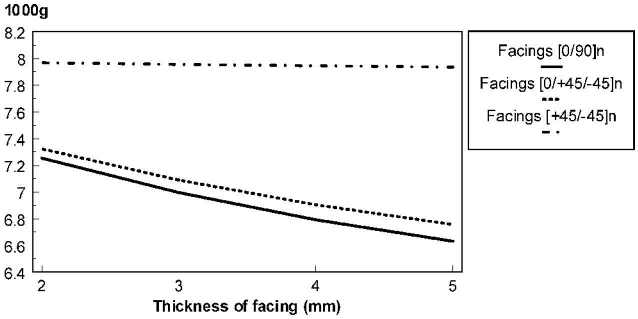

Fig. 5 Effect of the thickness of E-glass/epoxy facings on the loss factor of sandwich beams.

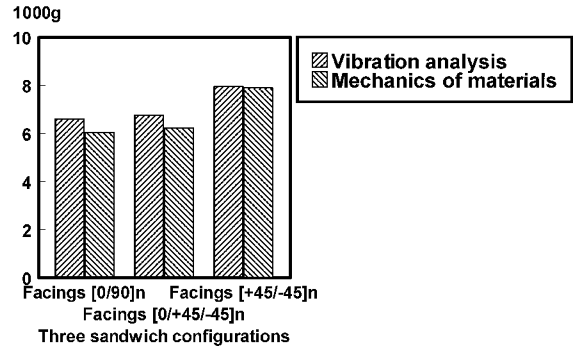

Fig. 6 Comparison between the loss factors of sandwich beams with E-glass/epoxy facings obtained by the methods based on free vibration analysis and mechanics of materials.

higher loss factor of the core. Indeed, in the case where the facing layup is $[+45/-45 \text{ deg}]_n$, the loss factors of the facings and core are close and variations in the facing thickness have a negligible effect on damping of the beam. In the case where the loss factor of the core is smaller than that of the facings, the tendency shown in Fig. 5 is reversed.

Finally, the loss factors of sandwich beams considered in Fig. 4 and obtained by the methods based on the analysis of free vibrations and mechanics of materials are compared in Fig. 6. As follows from this figure, the difference between the loss factors obtained by the two methods is small. Although the method based on mechanics of materials yields smaller values of the loss factor than the vibration method, the former method is sufficiently simple and reliable to be used for an estimate of the loss factor of sandwich beams.

Conclusions

The paper presents solutions of several problems involved in the analytical evaluation of damping in composite and sandwich structures. A general expression for the loss factor in a generally orthotropic lamina subject to axial stresses is derived in the paper. In addition, the loss factor of the lamina experiencing transverse shear is evaluated. Based on these findings, two methods of damping prediction in composite and sandwich structures are outlined. The first method is based on the analysis of the strain and dissipation energies during free vibrations of a simply supported symmetrically laminated composite or sandwich beam. The second method referred to as based on mechanics of materials predicts damping in a sandwich beam by considering its strain and dissipation energies as a result of an applied pressure. Both methods yield the loss factors dependent on the boundary conditions of the structure. The method based on the analysis of free vibrations explicitly reflects the effect of vibration frequency on the loss factor.

The results of the numerical analysis indicate that the loss factor of a lamina reaches a maximum value when the lamination angle is close to 45 deg. (If the lamina is narrow, i.e., the Poisson effect is negligible, the maximum loss factor corresponds to the lamination angle equal to 90 deg.) In the case of cross-ply laminates, the loss factor reduces as a result of adding longitudinal layers. The loss factor of sandwich beams considered in the examples was relatively insensitive to the mode of vibration. This means that damping is little affected by the frequency of motion, although the effect of the frequency on the loss factors of the fibers and matrix of the facings and the foam core may alter this conclusion. A decrease in the relative thickness of the facing resulted in a smaller damping (loss factor) of sandwich beams. This was due to a higher damping of the core, and if the relationship between the loss factors of the facings and core was reversed, the conclusion would be reversed as well. Finally, the loss factors of sandwich beams obtained by two methods suggested in the paper, i.e., the analysis of free vibrations and mechanics of materials, were compared. The difference between the methods was small in all configurations considered in the paper.

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